1.

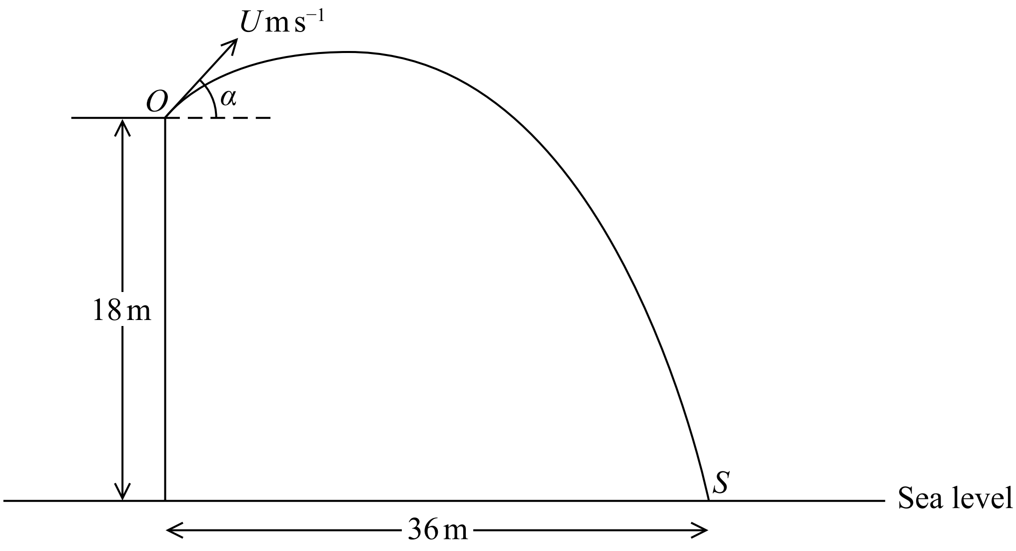


Figure 2

A boy throws a stone with speed $U\,\text{m s}^{-1}$ from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10\,\text{m s}^{-2}$

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

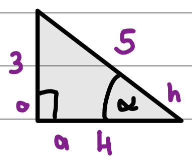
(5)

(c) Suggest two improvements that could be made to the model.

(2)

a) U is the initial Velocity that the stone was thrown at.

$\tan \alpha = 3/4$



SOHCAHTOA $\Rightarrow \cos\alpha = 4/5$, $\sin\alpha = 3/5$

We can split the motion of the stone into its vertical component and its horizontal component.

Horizontal : $S = 36m$ $\Rightarrow$ $S = ut$ 1

$u = U \cdot \cos\alpha$ $\quad 36 = U\cos\alpha \cdot t$ 1

$v =$ $\quad 36 = 4/5 \cdot Ut$

$a = 0\ ms^{-2}$ $\quad 45 = Ut \Rightarrow t = \frac{45}{u}$

$t = ?$

Vertical : $S = -18m$ $\quad S = ut + \frac{1}{2}at^2$ 1

$u = U\sin\alpha = U \cdot 3/5$ $\quad -18 = \frac{3}{5}U \cdot \frac{45}{U} + \frac{1}{2}(-10)\left(\frac{45}{u}\right)^2$ 1 1

$v = x$

$a = -10\ ms^{-2}$ $\quad -18 = 27 - \frac{10125}{u^2}$

$t = ?\ \frac{45}{u}$

$\Rightarrow 10125 = 45u^2$

$\Rightarrow U = \sqrt{\frac{10125}{45}} = 15\ ms^{-1}$ 1

$\Rightarrow U = 15ms^{-1}$

b) Vertical : $S = -18 + 10.8 = -7.2m$ $\quad v^2 = u^2 + 2as$

$u = U\sin\alpha = 15 \cdot 3/5 = 9ms^{-1}$ $\quad v^2 = (9)^2 + 2(-10)(-7.2)$ 1

$v = ?$ $\quad v = \sqrt{225}$

$a = -10\ ms^{-2}$ $\quad \Rightarrow v = 15ms^{-1}$ 1

Horizontal : $V = U\cos\alpha = 15 \times 4/5 = 12ms^{-1}$ 1

Go from two vector components of velocity to a scaler which is speed by finding magnitude of the velocity vector.

$\Rightarrow$ Speed $= \sqrt{15^2 + 12^2} = 19.209...$ 1 $\Rightarrow$ Speed when the stone is 10.8m above sea level will be $19ms^{-1}$ 1

c)

- take into account air Resistance 1
- what effect the wind has on the motion of the stone. 1

2.

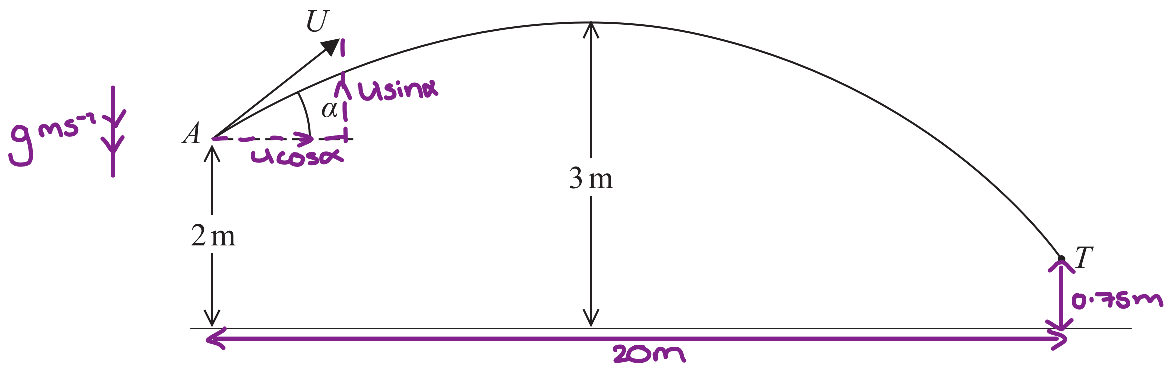


Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground.
The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2\alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

a) R(↑) - 1

$s = 3-2 = 1$
$u = U\sin\alpha$
$v = 0$
$a = -g$
$t = /$

$v^2 = u^2 + 2as$
$0 = (U\sin\alpha)^2 + 2(-g)(1)$
$0 = U^2\sin^2\alpha - 2g$
$U^2 = \dfrac{2g}{\sin^2\alpha}$ - 1

b) R(↑) - (1)

$s = -(2 - 0.75) = -1.25$

$u = U\sin\alpha$

$v = /$

$a = -g$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$-1.25 = (U\sin\alpha)t + \frac{1}{2}(-g)(t)^2$

$-1.25 = (U\sin\alpha)t - \frac{g}{2}t^2$ - (1) - (1)

R(→) - (1)

$s = 20$

$u = U\cos\alpha$

$v = /$

$a = 0$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$20 = (U\cos\alpha)t$ - (1)

$t = \frac{20}{U\cos\alpha}$ - (2)

(2) into (1)

$-1.25 = (U\sin\alpha)\left(\frac{20}{U\cos\alpha}\right) - \frac{g}{2}\left(\frac{20}{U\cos\alpha}\right)^2$ - (1)

$-1.25 = 20\tan\alpha - \frac{g}{2}\left(\frac{400}{U^2\cos^2\alpha}\right)$

$-1.25 = 20\tan\alpha - \frac{200g}{U^2\cos^2\alpha}$

// substitute $U^2 = \frac{2g}{\sin^2\alpha}$

$-1.25 = 20\tan\alpha - \frac{200g}{\left(\frac{2g}{\sin^2\alpha}\right)\cos^2\alpha}$ - (1)

$-1.25 = 20\tan\alpha - \frac{200g}{\frac{2g}{\tan^2\alpha}}$

$-1.25 = 20\tan\alpha - 100\tan^2\alpha$ - (1)

$100\tan^2\alpha - 20\tan\alpha - 1.25 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\tan\alpha = \frac{20 \pm \sqrt{(-20)^2 - 4(100)(-1.25)}}{2(100)}$ - (1)

$\tan\alpha = \frac{20 \pm \sqrt{900}}{200}$

$\tan\alpha = \frac{1}{4}$ $\qquad$ $\tan\alpha = -\frac{1}{20}$

$\alpha = 14.0°$ (3sf) $\qquad$ $\alpha = \cancel{-2.86°}$ (3sf)

$\alpha = 14.0°$ (3s.f.) - (1)

c) The target has dimensions - there will be a range of possible values for α. - 1

or:

-> Air resistance on ball

-> Wind effects

-> Ball has dimensions.

d) $u^2 = \frac{2g}{\sin^2\alpha}$ ← (use equation from a))

$u^2 = \frac{2g}{\sin^2 14.0}$ - 1

$u = 18.3 \text{ ms}^{-1}$ (3 s.f.)

$t = \frac{20}{u\cos\alpha}$ - 1 ← (equation from b))

$t = \frac{20}{18.3\cos 14.0}$

$t = 1.13$ seconds (3 s.f.) - 1

3.

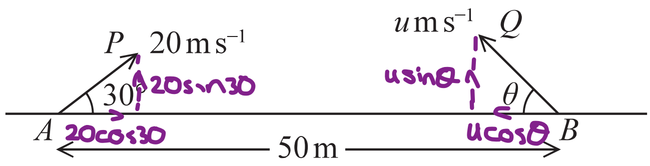


Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed 20 m s^{-1} at $30°$ to AB.

Ball Q is projected from B with speed $u\text{ m s}^{-1}$ at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

↳ R(↓)
a = g

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

a) R(↑)

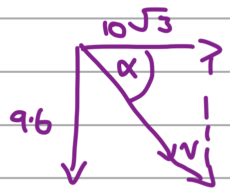
$s = /$
$u = 20\sin 30$
$v = v_1$
$a = -g$ – 1
$t = 2$

R(→)

$s = /$
$u = 20\cos 30$ – 1
$v = v_2$
$a = 0$
$t = 2$

$v = u + at$
$v_1 = 20\sin 30 - g(2)$
$v_1 = -9.6\text{ ms}^{-1}$ – 1

$v = u + at$
$v_2 = 20\cos 30$
$v_2 = 10\sqrt{3}\text{ ms}^{-1}$



$V = \sqrt{(9.6)^2 + (10\sqrt{3})^2}$
$V = 19.8\text{ ms}^{-1}$ (3s.f.) – 1

a) $\tan\alpha = \frac{9.6}{10\sqrt{3}}$ – 1

$\alpha = 29.0°$ (3 s.f.)

Velocity is $19.8\,ms^{-1}$ (3s.f.) at $29.0°$ (3s.f) below the horizontal – 1

b) for P:

$R(\uparrow)$

$s = ?$

$u = 20\sin 30$

$v = /$

$a = -g$

$t = 2$

$s = ut + \frac{1}{2}at^2$

$s = 2(20\sin 30) + \frac{1}{2}(-g)(2)^2$

$s = 0.4\,m$

– 1

When P and Q collide, the vertical distance of each must be equal. so if $s = 0.4m$ for P (when $R(\uparrow)$), $s = 0.4m$ for Q too (when $R(\uparrow)$)

for Q:

$R(\uparrow)$

$s = 0.4$

$u = u\sin\theta$

$v = /$

$a = -g$

$t = 2$

$s = ut + \frac{1}{2}at^2$

$0.4 = 2u\sin\theta + \frac{1}{2}(-g)(2)^2$

$0.4 = 2u\sin\theta - 2g$

$u\sin\theta = 0.2 + g$ – 1 – 1

for P:

$R(\rightarrow)$

$s = ?$

$u = 20\cos 30$

$v = /$

$a = 0$

$t = 2$

$s = ut + \frac{1}{2}at^2$

$s = 2(20\cos 30)$

$s = 20\sqrt{3}\ m$

If the total distance $AB = 50m$, and the (horizontal) from A -> collision $= 20\sqrt{3}\,m$, then the distance from collision -> B must be $(50 - 20\sqrt{3})\,m$

b) for Q:

R(←)

$s = 50 - 20\sqrt{3}$ – ①

$u = u\cos\theta$

$v = /$

$a = 0$

$t = 2$

$s = ut + \frac{1}{2}at^2$

$50 - 20\sqrt{3} = 2u\cos\theta$

$u\cos\theta = 25 - 10\sqrt{3}$ –② –①

① ÷ ②

i) $\frac{\cancel{u}\sin\theta}{\cancel{u}\cos\theta} = \frac{0.2 + g}{25 - 10\sqrt{3}}$

$\tan\theta = \frac{0.2 + g}{25 - 10\sqrt{3}}$ – ①

$\theta = 52.5°$ (3s.f.)

ii) $u\cos 52.5 = 25 - 10\sqrt{3}$

$u = \frac{25 - 10\sqrt{3}}{\cos 52.5}$

$u = 12.6$ (3s.f.) – ①

c) The model doesn't take into account the fact that P and Q aren't actually particles. – ①

4.

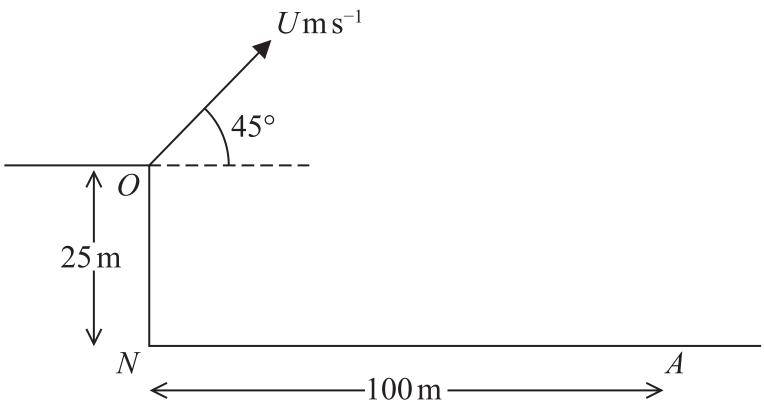


Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

↳ No air resistance.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

a)

Taking up as positive,

	Horizontal Comp	Vertical Comp
S	100	-25
u	$U\cos 45$	$U\sin 45$
v	$U\cos 45$	
A	0	$-g$
T		

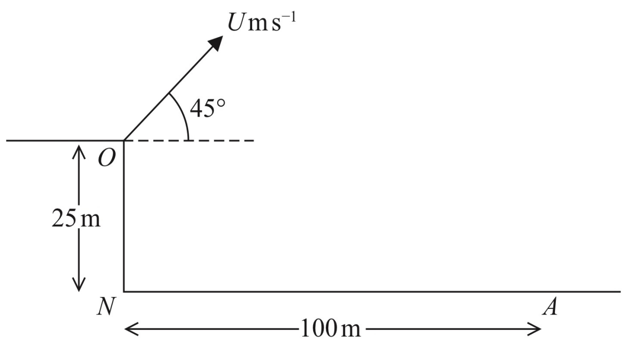


Figure 2

Using horizontal Motion

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}} \Rightarrow U\cos 45 = \frac{100}{t}$$

$$\hookrightarrow t = \frac{100}{U\cos 45}$$

Using Vertical motion

$$s = ut + \frac{1}{2}at^2$$

$$-25 = U\sin 45\, t - \frac{1}{2}gt^2$$

$$-25 = \frac{\cancel{U}\sin 45 \times 100}{\cancel{U}\cos 45} - \frac{1}{2}g\left(\frac{100}{U\cos 45}\right)^2$$

$$-25 = 100 \times \tan 45 - \frac{1}{2}g\left(\frac{100^2}{U^2\cos^2 45}\right)$$

$$-25 - 100 \times \tan 45 = -\frac{1}{2}g\left(\frac{10{,}000}{U^2\cos^2 45}\right)$$

$U = 28$ as required.

b)

$u = 28 \text{ ms}^{-1}$

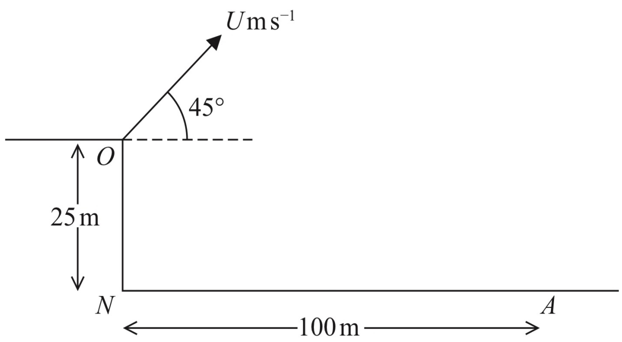


Figure 2

Using Vertical Motion

	Vertical Component
S	h
U	$28 \sin 45$
V	0
A	$-g$
T	

$v^2 = u^2 + 2as$

$0^2 = (28 \sin 45)^2 + 2(-g)(h)$

$0 = (28 \sin 45)^2 - 2gh$

$2gh = (28 \sin 45)^2$

$h = \dfrac{(28 \sin 45)^2}{2g} = 20\text{m}$

greatest height $= h + 25\text{m}$

$= 20 + 25 = 45\text{m}$

c)

New value of U > 28
air resistance causes a reduction in the final distance reached at a given velocity. ∴ To reach the same distance, a larger initial velocity is needed.

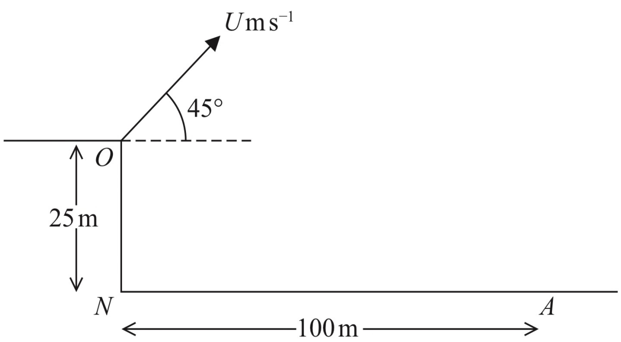


Figure 2

d)
more accurate value of g

Alternative Answers
→ Wind effect
→ Spin of the ball
→ Include size of the ball
→ Don't model ball as a particle
→ Consider shape of the ball.

5.

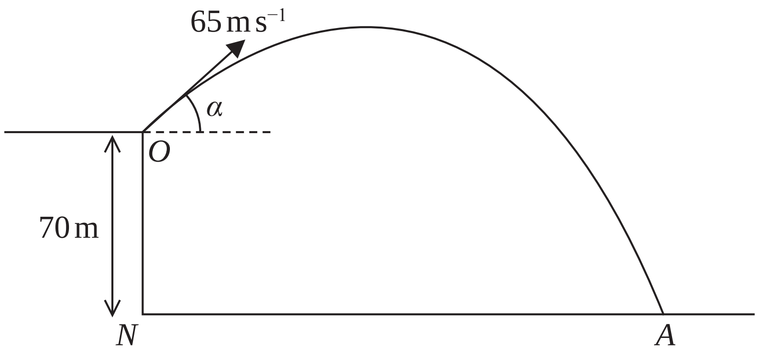


Figure 3

A small stone is projected with speed $65\,\text{m s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

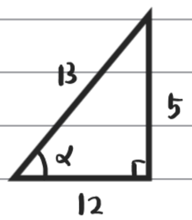
(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

(a)



$\tan\alpha = \frac{5}{12}$, $\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$

moving downwards, negative sign

1

$s = ut + \frac{1}{2}at^2$: $-70 = 65\sin\alpha \times t + \frac{1}{2} \times (-g) \times t^2$ 1

solve vertically

$$-70 = 65 \times \frac{5}{13} \times t - \frac{1}{2} \times 10 \times t^2$$

$$-70 = 25t - 5t^2$$

$$5t^2 - 25t - 70 = 0 \quad (1)$$

$$t^2 - 5t - 14 = 0$$

$$(t-7)(t+2) = 0$$

$$t = 7, -2$$ (the positive value is the only correct solution)

$$\therefore t = 7 \text{ seconds} \quad (1)$$

there is no acceleration horizontally

(b) $v = u + at$: $V_H = u_H + (0)t$

Solve horizontally the components at A to find V_H

$$V_H = u_H$$

$$= 65 \cos \alpha \quad (1)$$

$$= 65\left(\frac{12}{13}\right)$$

$$= 60 \text{ ms}^{-1} \quad (1)$$

$v = u + at$: $V_V = u_V - gt$

now solve vertically the components at A to find V_V

from our answer in (a)

$$= 65 \sin \alpha - 10 \times 7 \quad (1)$$

$$= 65\left(\frac{5}{13}\right) - 70$$

$$= -45 \text{ ms}^{-1}$$

$\therefore$ speed $= \sqrt{(v_H)^2 + (v_v)^2}$

$= \sqrt{(60)^2 + (-45)^2}$ 1

$= \sqrt{5625}$

$= 75 \text{ ms}^{-1}$ # 1

(c) An approximate value of g has been used. To make answers more reliable, use $g = 9.8 \text{ ms}^{-2}$ # 1